

A causal model for a closed universe

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We study a closed model of a universe filled with viscous fluid and quintessence matter components. The dynamical equations imply that the universe might look like an accelerated flat Friedmann-Robertson-Walker (FRW) universe at low redshift. We consider here dissipative processes which obey a causal thermodynamics. Here, we account for the entropy production via causal dissipative inflation.

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I. INTRODUCTION

Recent observational evidences suggest that the measured matter density of baryonic and dark matter is significantly less than one, i.e. its critical value. This implies that either the universe is open or that there are some other matter components which makes $\Omega_{total} \sim 1$. Combined measurements of CMB temperature fluctuations and of the distribution of galaxies on large scales strongly suggest the possibility of a flat universe [1, 2], which is consistent with the standard inflationary prediction[3]. Recent measurements of a Ia type distant supernova (SNe Ia) [4, 5], at redshift $z \sim 1$, indicate that the expansion of the present universe is accelerated.

An initial interpretation was to consider that in the universe there exists an important matter component that, in its most simple description, has the characteristic of the cosmological constant Λ , i.e. a vacuum energy density which contributes to a large component of negative pressure, and thus accelerates rather than decelerates the expansion of the universe. An alternative interpretation is to consider quintessence (or dark energy), in the form of a scalar field with a self-interacting potential. In the literature various examples of such Q a component have been considered starting from the pioneering paper [6], which also considered the case of cosmic strings. The non-minimally coupled scalar fields were considered in Ref. [7] and the scalar-tensor theories in Ref. [8]. The current status about dark energy can be found in Ref. [9].

On the other hand, in recent years important attention has been received by cosmological models which consider bulk viscosity. For instance, it was shown that the introduction of this kind of viscosity into cosmological models can avoid the big bang singularities [10, 11, 12], and any contribution from particle production may be modeled as an effective bulk viscosity [13]. The bulk viscosity typically arises in mixtures of different species (as in a radiative fluid)

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or of the same (but with different energies) fluids. The dissipation due to bulk viscosity converts kinetic energy of the particles into heat, and thus one expects it to reduce the effective pressure in an expanding fluid. This fact may play a crucial role in the inflationary era of the universe since it is interesting to know whether dissipative effects could be strong enough to make a large negative effective pressure leading to inflation.

Many, probably most, of the inflationary cosmological models considered are of a flat FRW type, since they are spatially isotropic and homogeneous. This implies that velocity gradients causing shear viscosity, and temperature gradients leading to heat transport, are absent. Thus any dissipation in an exact FRW universe is scalar, and therefore may be modeled as a bulk viscosity within a thermodynamical approach. These scalar dissipative processes may be treated in cosmology via the theory of general relativity, considering the bulk viscosity [14, 15], which is compatible with the homogeneity and isotropy assumptions for the universe. These dissipative processes may play an important role in the early universe, especially before nucleosynthesis [16]. Then we shall use the causal thermodynamical theory for processes not in equilibrium [17]. The stable and causal thermodynamics of Israel and Stewart replaces satisfactorily the unstable and non-causal theory of Eckart [18] and Landau and Lifshitz [19].

It is well known that the inflationary universe model is necessary for solving most of the cosmological puzzles, in the way that it explains how a large class of initial states can evolve into a unique final state that is consistent with our observed universe. In most of inflationary FRW cosmological models not only the scale factor $a(t)$ presents a positive acceleration $\ddot{a} > 0$, but also it is assumed from the very beginning that the geometry of the universe is completely flat.

In the light of this approach an interesting question to ask is whether this flatness may be due to a sort of compensation among different components that enter into the dynamical equations. In this respect, one of our goals is to address this question in a frame of a simple model. In the literature we find some descriptions along these lines. For instance, a closed model has been studied with an important matter component whose equation of state is given by $p = -\rho/3$. There, the universe expands at a constant speed [20]. Other authors, while using the same astronomically observed properties for the universe, have added a nonrelativistic matter density in which the total matter density Ω_0 is less than one, thus describing an open universe [21]. Also, a flat decelerating universe model has been simulated [22]. The common fact to all of these models is that, even though the starting geometry were other than that corresponding to the critical geometry, i.e. a flat geometry; all these scenarios are, at low redshift, indistinguishable from a flat geometry.

In this paper we wish to consider a closed universe model composed of two matter components: one related to a viscous matter and the other to quintessence matter. The geometry, together with these matter components, confabulates in such a special way that it gives rise to a flat accelerating universe scenario. In this regard, we use the standard ansatz (see Eqs. (16), (14) and (24) below) which introduce five parameters in our model, where there exists a range of these parameters that gives an appropriate value for the entropy of the universe. The value obtained for the entropy in our model agrees with the expected value obtained from usual inflationary re-heating. In this way, as we will see, our model accounts for the generally accepted entropy production, via warm inflation [23]. In this kind of model the radiation is continuously produced by the decay of the inflaton scalar field. In this way this field should be coupled to ordinary matter and, contrary to usual inflation, primordial density fluctuations are originated much more from thermal fluctuations than from quantum fluctuations of the inflaton [24]. At the end, the parameters of the resulting model could be estimated by using astronomical data.

The outline of the present paper is as follows: In Sec. II we briefly review the system of gravitational field equations and the transport equation for bulk viscosity. In Sec. III and IV a power law FRW and de Sitter FRW causal cosmologies are considered. In Sec. V some conclusions are given.

II. THE SYSTEM OF GRAVITATIONAL FIELD EQUATIONS AND THE TRANSPORT EQUATION FOR BULK VISCOSITY

Now we shall obtain the field equations for the considered cosmological viscous model. The energy-momentum tensor of the fluid with a bulk viscosity is given by

$$T_{\mu\nu} = (\rho_M + p_M + \pi)v_\mu v_\nu + (p_M + \pi)g_{\mu\nu}, \quad (1)$$

where ρ_M and p_M are the thermodynamical density and the pressure of the fluid, while π is the bulk viscous pressure, and $v^\mu = \delta_0^\mu$.

The contribution to the energy-momentum tensor related to the classical scalar field ϕ , which is minimally coupled to gravity, becomes given by:

$$T_{\mu\nu} = \phi_{,\mu}\phi_{,\nu} - g_{\mu\nu} \left(\frac{1}{2}\phi_{,\alpha}\phi^{,\alpha} + V(\phi) \right), \quad (2)$$

where $V(\phi)$ is the scalar potential.

Now we consider the FRW metric

$$ds^2 = -dt^2 + a^2(t) \left(\frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2) \right). \quad (3)$$

From Eq. (1) and (2) we have the gravitational field equations:

$$3H^2 + 3\frac{k}{a^2} = \kappa \left(\frac{1}{2}\dot{\phi}^2 + V + \rho_M \right) \quad (4)$$

$$2\dot{H} + 3H^2 + \frac{k}{a^2} = \kappa \left(-\frac{1}{2}\dot{\phi}^2 + V - p_M - \pi \right) \quad (5)$$

where $H = \frac{\dot{a}(t)}{a(t)}$ and k is the curvature parameter and $\kappa = 8\pi G$, with G the Newtonian gravitational constant.

The fluid conservation equation is given by

$$\dot{\rho}_M + 3H(\rho_M + p_M + \pi) = 0 \quad (6)$$

and the equation for the scalar field by

$$\ddot{\phi} + 3H\dot{\phi} + \frac{\partial V}{\partial \phi} = 0. \quad (7)$$

The energy density associated with the scalar field is given by

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi) \quad (8)$$

and the pressure by

$$p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi). \quad (9)$$

The scalar field has a state equation defined by $p_\phi = w\rho_\phi$ and satisfies the conservation equation

$$\dot{\rho}_\phi + 3H(\rho_\phi + p_\phi) = 0. \quad (10)$$

From Eq. (4) and the weak energy condition $\rho_\phi \geq 0$ we conclude that, for desired flatness simulation, i.e. Eq. (4) becoming the well known Friedmann Equation for a flat FRW universe

$$3H^2 = \kappa\rho_M, \quad (11)$$

we must take $k = 1$ in the field equations. Thus

$$\kappa\rho_\phi = 3/a^2. \quad (12)$$

From here we have that $\kappa\dot{\rho}_\phi = -3\dot{a}/a^3 = -6H/a^2$ and, substituting into Eq. (10), obtain that $w = -\frac{1}{3}$. Notice that this result is independent of the explicit expression for the scale factor as a function of t .

At this point we should notice that our model is intrinsically closed, therefore we should stress that our model is completely different with the flat standard Einstein model which characterizes with $\Omega_M = 1$ and is ruled out by the current astronomical observations. In fact, the curvature term (which is intrinsically geometric) cannot be eliminated by a simple redefinition of the scalar field ϕ , since the curvature is a geometrical property, which follows directly from the metric tensor and which enters into the FRW line elements. These two models become indistinguishable only for low redshifts and are similar to the cases studied in Refs. [21, 22, 25].

Another point that we would like to stress here is that almost all physical quantities are obtained in terms of the scale factor and its derivatives. From Eq. 12 we get ρ_ϕ as a function of a . Since $w = -1/3$, then $\dot{\phi}^2 = V(\phi) = 2/\kappa a^2$. Also, from Eq. 5 we get that $\kappa\pi = -23\dot{H}^2 - 3\gamma H^2$, where we have used the state of equation $p_M = (\gamma - 1)\rho_M$ for the matter component and Eq. 11. As we will see and in order to obtain an explicit expression for the scalar potential V as a function of the scalar field ϕ , we need the temporal dependence of the scale factor. In this respect we will take a power law and an exponential inflationary solutions.

From now on we shall use the simplest case of scalar dissipation due to bulk viscosity ξ , modeling cosmological dissipation from the consideration of nonequilibrium thermodynamics via Israel-Stewart theory. The bulk viscous pressure π is given by the transport equation (linear in π):

$$\tau \dot{\pi} + \pi = -3H\xi - \frac{1}{2}\tau\pi \left(3H + \frac{\dot{\tau}}{\tau} - \frac{\dot{\xi}}{\xi} - \frac{\dot{T}}{T} \right), \quad (13)$$

where τ is the relaxation time (which removes the problem of infinite propagation speeds), ξ the coefficient of bulk viscosity and T is the temperature of the fluid. In the non-causal formulation $\tau = 0$ and then Eq. (13) has a simple form $\pi = -3H\xi$.

Following [14, 26] we shall take the different thermodynamic quantities to be simple power functions of the density ρ_M ,

$$\xi = \alpha \rho_M^m, \quad T = \mu \rho_M^r \quad \text{and} \quad \tau = \frac{\xi}{\rho_M} = \alpha \rho_M^{m-1} \quad (14)$$

with α, μ, m and r positive constants. The expression for τ is used as a simple procedure to ensure that the speed of viscous pulses does not exceed the speed of light. For an expanding cosmological model the constant m should be positive and

$$\tau > H^{-1} \quad (15)$$

in order to have a physical behaviour for the coefficient of viscosity ξ and the relaxation time τ [14, 27].

III. POWER LAW FRW CAUSAL INFLATIONARY COSMOLOGY

Now we are interested in power law inflationary models; thus we shall take the scale factor in the form

$$a(t) = a_0 \left(\frac{t}{t_0} \right)^n, \quad (16)$$

where a_0 and n are constant parameters, with $n > 1$. Then, using Eq. (16) we obtain

$$H = \frac{n}{t}, \quad (17)$$

$$\kappa \rho_\phi = \frac{3}{a_0^2} \left(\frac{t_0}{t} \right)^{2n} \quad (18)$$

and

$$\kappa p_\phi = -\frac{1}{a_0^2} \left(\frac{t_0}{t} \right)^{2n} \quad (19)$$

for the Hubble constant, mass density and pressure of the field ϕ respectively.

In this case the solution for the scalar field and its potential are given by

$$\phi(t) = \phi_0 \left(\frac{t_0}{t} \right)^{n-1} \quad (20)$$

and

$$V(\phi) = V_0 \left(\frac{\phi}{\phi_0} \right)^{\frac{2n}{n-1}}, \quad (21)$$

respectively, where $\phi_0 = \phi(t_0) = \pm \frac{\sqrt{2/\kappa} t_0}{1-n}$ and $V_0 = \frac{2}{\kappa a_0^2}$.

We shall consider for the fluid a barotropic equation of state

$$p_M = (\gamma - 1)\rho_M, \quad (22)$$

where $0 \leq \gamma \leq 2$.

Then for the power law expansion (16) and from eqs. (6), (13) and (14) (with the standard thermodynamic relation for the temperature of a barotropic fluid $r = (\gamma - 1)/\gamma$) we obtain

$$\ddot{\rho}_M + (3n + 1) \frac{\dot{\rho}_M}{t} + \frac{1}{\alpha} \rho_M^{1-m} \dot{\rho}_M + \frac{3n\gamma}{\alpha t} \rho_M^{2-m} - 9n^2 \left(1 - \frac{\gamma}{2}\right) \frac{\rho_M}{t^2} - \frac{(2\gamma - 1)}{2\gamma} \frac{\dot{\rho}_M^2}{\rho_M} = 0. \quad (23)$$

Note that this equation may also be written in terms of H using Eq. (11).

Case $m \neq 1$: For solving Eq. (23) we shall take the following anzats [27]:

$$\rho_M = \rho_M^0 \left(\frac{t}{t_0}\right)^{\left(\frac{-1}{1-m}\right)}. \quad (24)$$

For physical sense we must have $m < 1$. Inserting (24) into Eq. (23) we obtain, for $m \neq 1$, that the constant ρ_M^0 is given by

$$\rho_M^0 = \left(\frac{\alpha}{t_0(1-m)}\right) \times \left(\frac{(2\gamma)^{-1} - 3n(1-m) - 9n^2(1-\gamma/2)(1-m)^2}{(1-3n\gamma(1-m))}\right)^{1/(1-m)}. \quad (25)$$

Later on we shall assume that the parameters γ , n , m and ρ_M^0 are given; then we can find the parameter α . The pressure p is obtained by using the equation of state (22).

Now from Eq. (22) and Eq. (6) we obtain for the bulk viscous pressure

$$\pi = -\frac{\dot{\rho}_M}{3H} - \gamma\rho_M \quad (26)$$

and substituting (24) and p_M into (26) we obtain

$$\pi = -\left(\gamma - \frac{1}{3n(1-m)}\right) \rho_M^0 \left(\frac{t}{t_0}\right)^{-1/(1-m)}. \quad (27)$$

From here we see that $\pi < 0$ if $\gamma > 1/(3n(1-m))$, $\pi > 0$ if $\gamma < 1/(3n(1-m))$ and $p = 0$ if $m = 1 - 1/(3n\gamma)$.

The entropy and the temperature for local equilibrium satisfy the Gibbs equation

$$TdS = (\rho_M + p_M)d\left(\frac{1}{N}\right) + \frac{1}{N}d\rho_M \quad (28)$$

where N is the number density, and satisfy the conservation equation

$$\dot{N} + 3HN = 0 \quad (29)$$

from which we get the solution

$$N(t) = N_0 \left(\frac{t}{t_0}\right)^{-3n} \quad (30)$$

or equivalently $Na^3(t) = \text{const.}$

Using (28) and (30) we obtain the well known evolution equation for the entropy (neglecting the heat flux and the shear viscosity):

$$\dot{S} = -\frac{3H\pi}{NT}. \quad (31)$$

From eqs. (17), (14), (24), (27) and (30) we have

$$\dot{S} = \frac{3n\gamma - 1/(1-m)}{N_0\mu t_0} (\rho_M^0)^{1/\gamma} \left(\frac{t}{t_0}\right)^{(3n-1)-1/\gamma(1-m)}. \quad (32)$$

For a reasonable physical behaviour we must satisfy $\dot{S} > 0$, which implies the condition $m < 1 - 1/\gamma(3n - 1)$.

The total entropy Σ in a comoving volume is defined by $\Sigma = SNa^3(t)$. Then, by considering Eqs. (30) and (31) we may write the growth of total nondimensional comoving entropy over a proper time interval from t_i until t_f as [14]:

$$\Sigma_f - \Sigma_i = -\frac{3}{k_B} \int_{t_i}^{t_f} \frac{\pi H a^3(t)}{T} dt. \quad (33)$$

Now taking respectively t_i and t_f as the beginning and the exit time for the inflation period of the universe, and from eqs. (16), (17), (14), (24), (27) and (33), we obtain for the increase in total nondimensional entropy in the comoving volume $a^3(t)$ the following expression:

$$\Sigma_f - \Sigma_i = \frac{\gamma a_0^3 (\rho_M^0)^{1/\gamma}}{k_B \mu} \times \left[\left(\frac{t_f}{t_0} \right)^{3n - \frac{1}{\gamma(1-m)}} - \left(\frac{t_i}{t_0} \right)^{3n - \frac{1}{\gamma(1-m)}} \right]. \quad (34)$$

The typical values for the beginning and ending times of inflation are $t_i \approx 10^{-35}$ s and $t_f \approx 10^{-32}$ s respectively. In our following numerical considerations we shall take the reference time t_0 equal to the ending one, i.e. $t_0 = t_f = 10^{-32}$ s. Considering that the universe at the end of the inflation period exits to the radiation era, we can constrain some of the constants of integration of the above formulae. Effectively, we know that the temperature of the universe at the beginning of the radiation era was $T = 10^{14} \text{ GeV} = 1.16 \times 10^{27} \text{ K}$ [29]. Then we have that the temperature of the end of inflation must be $T_f = 1.16 \times 10^{27} \text{ K}$. On the other hand we know that during this radiation period $\rho = a_r T^4$ is valid, where $a_r = \frac{\pi^2 k_B^4}{15 c^3 \hbar^3} = 7.56 \times 10^{-15} \text{ J m}^{-3} \text{ K}^{-4}$. From this we conclude that at the end of the inflation period (or at the beginning of the radiation era) we have $\rho_f \approx 10^{93} \text{ J m}^{-3}$. This implies that in Eq. (14) $r = 1/4$ for the temperature, i.e. $\gamma = 4/3$.

Then we can summarize these typical values for the inflation period as [14, 30, 32]:

$$\begin{aligned} t_i &\approx 10^{-35} \text{ s}; \quad t_f \approx 10^{-32} \text{ s}; \quad a_i \approx c t_i, \\ T_f &\approx 10^{27} \text{ K}, \quad \gamma = 4/3, \quad \rho \approx \times 10^{93} \text{ J/m}^3. \end{aligned} \quad (35)$$

The e-folding parameter $Z = \ln[a(t_f)/a(t_i)]$ for the power law inflation (16) takes the form

$$Z = n \ln \left(\frac{t_f}{t_i} \right). \quad (36)$$

It is well known that for solving the problems of the standard model in cosmology we must have $Z \approx 60 - 70$. Thus from (14), (34), (35) and (36) we have

$$\Sigma_f - \Sigma_i \approx \frac{4a_i e^{3Z} (\rho_M^f)^{3/4}}{3k_B \mu} \times 10^{26} \left(1 - (10^{-3})^{3n - \frac{3}{4(1-m)}} \right), \quad (37)$$

where we have considered the relation $a_f^3 = a_i^3 e^{3Z}$ following from the definition of Z . We see from Eq. (37) that, if the inequality $3n\gamma(1 - m) \gg 1$ (as we have obtained above) is satisfied (in this case $\Sigma_f \gg \Sigma_i$), we obtain the accepted value for the total entropy in the observable universe [29, 30]

$$\Sigma \approx 10^{88}. \quad (38)$$

Thus this model can account for the generally accepted entropy production (38) via causal dissipative inflation, without re-heating.

Thus, we describe a dissipative accelerating period for the universe which resembles a warm inflationary scenario. To see this more clearly let us consider eq.(6). Introducing the scalar inflaton field χ in such a way that we define $\rho_M \equiv \frac{1}{2}\dot{\chi}^2 + V(\chi)$ and $p_M \equiv \frac{1}{2}\dot{\chi}^2 - V(\chi)$, where $V(\chi)$ is the effective potential associated with the inflaton field, we obtain the following expression:

$$(3H + \Gamma)\dot{\chi} + V'(\chi) = 0, \quad (39)$$

where we have introduced Γ such that $\Gamma\dot{\chi} = 3H\pi/\dot{\chi}$. This term describes the density energy dissipated by the χ field into the thermalized bath. We have also used the slow-roll-over condition given by $\ddot{\chi} \ll (3H + \Gamma)\dot{\chi}$ (or equivalently,

$\dot{\rho}_M \ll +3H(\gamma\rho_M + \pi)/(\gamma - 1)$). Expression (39) is the basic equation which describes a warm inflationary universe model [23]. One of the interesting characteristics of this model is does not need an intermediate reheating period as in usual inflationary models, since the transformation of vacuum energy into radiation energy occurs throughout the inflationary period and thus the inflationary epoch smoothly terminates into a radiation dominated regime[31].

At the end of the inflationary period, when the universe enters into the radiation dominated period, we have found that the growth of entropy which results is $\Sigma_f \approx 10^{88}$. This result gives an answer to the total entropy problem, whose question is why the total entropy in the observable part of the universe is so large. Certainly this problem is one of the various “puzzles” found in the standard Big-Bang cosmological model.

A special case is obtained here when the parameter $m = 1/2$. In this case we obtain a constraint for each and all of the constants except ρ_M^0 . Effectively, substituting the expression $\rho_M = \rho_M^0 (t_0/t)^2$ into Eq. (23) we obtain

$$\alpha = \frac{\sqrt{3/\kappa} n(3n\gamma - 2)}{9(1 - \gamma/2)n^2 + 6n - 2/\gamma}. \quad (40)$$

Now the relaxation time should satisfy the condition (15). But $\tau = \alpha \rho_M^{1/2}$ and considering (11) we get $\alpha < \sqrt{3/\kappa}$. This finally implies that

$$n \gg 1/\gamma, \quad (41)$$

which is satisfied for a power law inflationary universe, since there n is greater than one. We could get an estimation of the parameter n by using the result (38) together with Eq. (37) in which $m = 1/2$. We get that $n \gtrsim 1.6$ for $Z \lesssim 70$.

Case $m = 1$: For this value of m we have to start from Eq. (23) which gives the solution [27]

$$\rho_M = \rho_M^0 e^{-2\gamma t/\alpha} \left(\frac{t}{t_0} \right)^{-\gamma(3n+2)}, \quad (42)$$

where $n^2 = 2\gamma/9$ and ρ_M^0 is an arbitrary constant. Unfortunately this solution implies a decreasing entropy

$$S(t) = \frac{\gamma}{\mu_0 N_0} \left(\rho_M^0 \right)^{1/\gamma} \left(\frac{t_0}{t} \right)^2 e^{-2t/\alpha} + const \quad (43)$$

which is a non-physical behaviour.

IV. DE SITTER FRW CAUSAL INFLATIONARY MODEL

We also can consider the de Sitter kind of inflation models, for a closed FRW with flat dynamics. In this case the scale factor is given by $a(t) = e^{H_0 t}$, where $H_0 = const$, and the solution for the scalar field and its potential is obtained from Eqs.(8), (9) and (12) and is given by

$$\phi(t) = \phi_0 e^{-H_0 t} \quad (44)$$

and

$$V(\phi) = V_0 \left(\frac{\phi}{\phi_0} \right)^2, \quad (45)$$

respectively, where $\phi_0 = \pm \sqrt{2/\kappa}/H_0$ and $V_0 = 2/\kappa$.

Now we have $H = H_0$ or equivalently, by Eq. (11), $\rho_M = const$. Thus from (6) we obtain that the viscous pressure is given by $\pi = -\gamma \rho_M$ and then from (31) we see that always $\dot{S} \geq 0$. Now Eq. (13) yields $H = \gamma \rho_M^{1-m}/(3\alpha(1 - \gamma/2))$ and considering Eq. (11), we have for $m \neq 1/2$:

$$H = H_0 =: \left[\frac{3^m \alpha (2 - \gamma)}{2\kappa^{m-1} \gamma} \right]^{1/(1-2m)}, \quad (46)$$

with

$$\xi = \xi_0 = \alpha \left(\frac{3}{\kappa} \right)^m H_0^{2m}, \quad T = T_0 = \mu \left(\frac{3}{\kappa} H_0^2 \right)^{(\gamma-1)/\gamma}. \quad (47)$$

For $m = 1/2$ we conclude that $H = \tilde{H}$, where \tilde{H} is an arbitrary constant and

$$\alpha = \frac{2\gamma}{\sqrt{3\kappa}(2-\gamma)}. \quad (48)$$

Here

$$\xi = \alpha \sqrt{\frac{3}{\kappa}} \tilde{H}, \quad T = T_0 = \mu \left(\frac{3}{\kappa} \tilde{H}_0^2 \right)^{(\gamma-1)/\gamma}. \quad (49)$$

It is easy to show that in both cases, i.e. for any value of m , we have that the relaxation time is given by

$$\tau = \frac{2\gamma}{3H(2-\gamma)}. \quad (50)$$

Then from Eq. (15) we have

$$\gamma > \frac{6}{5} \quad (51)$$

in accordance with the results obtained in [14].

Now considering that $a(t) = e^{Ht}$, $Z = H(t_f - t_i)$ and from (33) we obtain

$$\Sigma_f - \Sigma_i = \frac{\gamma a_i^3 \rho_M^i}{k_B T} (e^{3Z} - 1), \quad (52)$$

where c has been restored. From $Z \approx 60$ and (35) we have that $\Sigma_f \gg \Sigma_i$ and obtain that $\Sigma_f \approx 10^{88}$ in agreement with the accepted value at the end of the period of inflation.

V. CONCLUSIONS

In this paper we have described a closed model universe composed of a viscous matter and a dark energy components. The dark energy together with the curvature term “confabulate” so that they result in the dynamical Friedmann Equation for a flat FRW universe, as is expressed by Eq. (11). On the other hand, the viscous matter component gives rise to an inflationary universe model at non-vanishing temperature.

We have used the anzats (16), (14) and (24) which introduce the following five parameters: n , α , m , r and μ . There exists a range of these parameters that gives an appropriate value for the entropy of the universe. In fact we saw that $n > 1$, $r = (\gamma - 1)/\gamma$, where γ is the equation of state parameter, $m < 1 - 1/\gamma(3n - 1)$ for $m \neq 1$ and α being an arbitrary constant. For a given value of α we would get a value of ρ_M^0 by using Eq. (25). From these ranges we have gotten an appropriate value for the entropy of the universe $\Sigma \approx 10^{88}$. We could say the same for $m = 1/2$, where now α is given by Eq. (40), which should satisfy $\alpha \ll \sqrt{3/\kappa}$ and $n \gg 1/\gamma$. Notice that, in this latter case, the matter component corresponds to quintessence, since the pressure becomes negative. Thus, the value obtained for the entropy in our model agrees with the expected value obtained from usual inflationary re-heating, which accounts for the generally accepted entropy production, via warm inflation[23]. The radiation period, after inflation, is continuously produced by the decay of the inflaton scalar field. In this way this field should be coupled to ordinary matter and, contrary to usual inflation, primordial density fluctuations are originated from thermal fluctuations rather than from quantum fluctuations of the inflaton[24].

Our results have been determined both for power-law (i.e. $a \sim t^n$, $n > 1$) and for de Sitter (i.e. $a \sim e^{H_0 t}$, $H_0 = \text{const.}$) inflationary universe models.

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